The shapes of level curves of real polynomials near strict local minima

Miruna-Ştefana Sorea

SISSA - Scuola Internazionale Superiore di Studi Avanzati, Trieste, Italy
and
RCMA Lucian Blaga University Sibiu, Romania
e-mail: msorea@sissa.it

Abstract: Consider a real bivariate polynomial function that has a strict local minimum at the origin and that vanishes at this point. In a sufficiently small neighborhood of the origin, the non-zero level curves of this function are smooth Jordan curves. Whenever the origin is a Morse strict local minimum, the small enough level curves become boundaries of convex topological disks. Otherwise, the levels may be non-convex, as it was proven by M. Coste.

In order to measure this non-convexity, we introduce a combinatorial object called the Poincaré-Reeb tree associated to a level curve and to a projection direction. Our main goal is to characterize all topological types of Poincaré-Reeb trees. I will explain how to construct a family of polynomials that realizes a large class of these trees.

To this end, we first reduce the problem to the univariate setting, via the interplay between the real polar curve and the discriminant curve. Using a combinatorial tool inspired by É. Ghys' recent work, we give a new and constructive proof for the existence of univariate Morse polynomials whose graphs have preassigned shapes, encoded by separable "Arnold snakes".

References

[1] V. I. ARNOLD, Snake calculus and the combinatorics of the Bernoulli, Euler and Springer numbers of Coxeter groups. *Uspekhi Matematicheskikh Nauk 47.1(283)(1992), pp. 3-45, 240.*

[2] É. GHYS, A singular mathematical promenade. *ENS Éditions, Lyon, 2017. viii+302 pp.*

[3] M.-Ş. SOREA, The shapes of level curves of real polynomials near strict local minima. *PhD thesis, 2018, Université de Lille, France.* `https://tel.archives-ouvertes.fr/tel-01909028/document`

[4] M.-Ş. SOREA, Constructing separable Arnold snakes of Morse polynomials. *Portugaliae Mathematica 77 (2020), no. 2, 219-260.*

[5] M.-Ş. SOREA, Measuring the local non-convexity of real algebraic curves. *Journal of Symbolic Computation 109 (2022), 482-509.*

[6] M.-Ş. SOREA, Permutations encoding the local shape of level curves of real polynomials via generic projections. To appear in *Annales de l'Institut Fourier.*