

# Tropical Differential Geometry

*Zeinab Toghani*

Los Andes University

**Abstract:** Let  $I$  be an ideal of the ring of Laurent polynomials  $K[x_1^{\pm 1}, \dots, x_n^{\pm 1}]$  with coefficients in a real-valued field  $(K, \nu)$ . The fundamental theorem of tropical algebraic geometry states the equality  $\text{trop}(V(I)) = V(\text{trop}(I))$  between the tropicalization  $\text{trop}(V(I))$  of the variety  $V(I) \subset (K^*)^n$  and the tropical variety  $V(\text{trop}(I))$  associated to the tropicalization of the ideal  $I$ .

In this talk, I will speak about tropical differential geometry and I will show the above result for a differential ideal  $J$  of the ring of differential polynomials  $K[[t]]\{x_1, \dots, x_n\}$ , where  $K$  is an uncountable algebraically closed field of characteristic zero.

I define the tropicalization  $\text{trop}(\text{Sol}(J))$  of the set of solutions  $\text{Sol}(J) \subset K[[t]]^n$  of  $J$ , and the set of solutions  $\text{Sol}(\text{trop}(J)) \subset (\mathcal{P}(\mathbb{Z}_{\geq 0})^n)$  associated to the tropicalization of the ideal  $J$ .

I will show the equality  $\text{trop}(\text{Sol}(J)) = \text{Sol}(\text{trop}(J))$ . Later I will extend the definitions for an ideal in the ring  $K[[t_1, \dots, t_m]]\{x_1, \dots, x_n\}$ .